PHYSICAL REVIEW E **68**, 067402 (2003)

Dust size distribution for dust acoustic waves in a magnetized dusty plasma

Wen-shan Duan[1]* and John Parkes[2]
[1]*Department of Physics, Northwest Normal University, Lanzhou 730070, China*
[2]*Department of Mathematics, University of Strathclyde, Glasgow G1 1XH, United Kingdom*
(Received 6 May 2003; revised manuscript received 11 July 2003; published 30 December 2003)

A reasonable normalization for a magnetized dusty plasma with many different dust grains is adopted, which varies self-consistently with the system parameters. A Zakharov-Kuznetsov equation for small but finite amplitude dust acoustic waves is obtained for magnetized dusty plasma which contains different dust grains by using the reductive perturbation technique. We study the dust size distribution. Some comparisons are made between dusty plasma in which the dust size distribution is considered, and the monosized dusty plasma in which there is only one kind of dust grain whose size is the average dust size. This suggests that both soliton velocity and width are larger than that for monosized dusty plasma, but its amplitude is smaller than that for monosized dusty plasma. If there are positively charged dust grains, compressive solitary waves may exist. The velocity, amplitude, and width of a soliton in multidimensional form for a magnetized dusty plasma which contains many different dust grains are studied as well.

DOI: 10.1103/PhysRevE.68.067402 PACS number(s): 52.35.Sb, 52.25.Vy, 05.45.Yv

I. INTRODUCTION

Dusty plasma has been studied extensively recently. Dusty plasma consists of ions, free electrons, and microsized particles. Dusty plasma is thought to play an important role in star and planet formation, such as planetary rings, cometary surroundings, interstellar clouds, and the lower parts of the earth's ionosphere [1,2]. In the laboratory, dust particles appear as impurities and can significantly influence the behavior of the surrounding plasma [3]. Recently the study of dusty plasma has developed rapidly with many applications in both laboratory and space plasma [4]. Dust acoustic waves (DAW) were first reported theoretically in unmagnetized dusty plasma by Rao *et al.* [5]. On the other hand, at higher frequency, Shukla and Silin showed the existence of dust ion acoustic waves (DIAW) [6]. Recent laboratory experiments on dusty plasmas have confirmed the existence of DAW and DIAW [7–9]. Furthermore, low-frequency electrostatic ion-acoustic and ion-cyclotron waves have been studied in a magnetized dusty plasma [10]. However, most of these theories have focused attention on dusty plasma containing only monosized dust grains [5,6,10]. In practice, the dust grains have many different sizes [11–14]. Several authors have studied the dust size distributions [15–20].

In this paper, the normalization for a magnetized dusty plasma with different dust grains is adopted. Furthermore, a Zakharov-Kuznetsov (ZK) equation is obtained which is similar to that for a monosized dusty plasma [21]. Some different results are obtained.

The paper is organized in the following fashion. In Sec. II we normalize the equations of motion for a dusty plasma with N different dust grains. In Sec. III we obtain a ZK equation. In Sec. IV we study how one encounters compressive DA solitary waves when the plasma contains both negative and positive dust grains. The amplitude, width, and propagation velocity of a soliton are studied as well. In Sec. V we study a multidimensional soliton.

II. NORMALIZATION AND GOVERNING EQUATION FOR A MAGNETIZED DUSTY PLASMA WITH N DIFFERENT DUST GRAINS

We now consider a three-component dusty plasma consisting of massive, negatively charged dusty fluid, Boltzmann distributed electrons, and ions in the presence of an external static magnetic field $\vec{B}=B\vec{k}$, where $\vec{k}$ is a unit vector along the z direction. Now we assume that there are N different dust grains with different dust size or mass. Thus, we have $n_{i0}=\sum_{j=1}^{N}Z_{dj}n_{dj0}+n_{e0}$, where n_{i0}, n_{dj0}, n_{e0} are the unperturbed ion, jth dust grain, and electron number density, respectively. Z_{dj} is the number of electrons residing on the jth dust grain. Let n_{dj}, m_{dj} represent the density and the mass, respectively, of the jth dust grain, ϕ is the electrostatic potential, and $\vec{u}_{dj}=u_{dj}\vec{i}+v_{dj}\vec{j}+w_{dj}\vec{k}$ is the jth dust grain velocity, where $\vec{i},\vec{j},\vec{k}$ are the unit vectors in the x,y, and z directions, respectively. We also let T_e and T_i stand for the temperatures of electrons and ions, respectively.

We now define the following quantities. The effective temperature is defined by $T_{eff}=T_iT_e/\mu T_e+\nu T_i$, where $\mu=n_{i0}/N_{tot}\overline{Z}_{d0}$, $\nu=n_{e0}/N_{tot}\overline{Z}_{d0}$. The total number density of all dust grains is $N_{tot}=\sum_{j=1}^{N}n_{dj0}$, where n_{dj0} is the unperturbed density of the jth dust grains. The average charged number residing on a dust grain is $\overline{Z}_{d0}=(\sum_{j=1}^{N}n_{dj0}Z_{dj0})/N_{tot}$. The average dust size or average dust radius is $\overline{a}=(\sum_{j=1}^{N}a_jn_{dj0})/N_{tot}$, where a_j is the radius of jth dust grains; here we assume that all the dust grains are spherical. The average mass of dust grains is $\overline{m}_d=\sum_{j=1}^{N}m_{dj}n_{dj0}/N_{tot}$. The effective Debye length is defined as $\lambda_{Dd}=(T_{eff}/4\pi\overline{Z}_{d0}N_{tot}e^2)^{1/2}$. The inverse of effective dusty plasma frequency is defined by $\omega_{pd}^{-1}=[\overline{m}_d/(4\pi N_{tot}\overline{Z}_{d0}^2e^2)]^{1/2}$. The effective dust acoustic speed is defined by $C_d=(\overline{Z}_{d0}T_{eff}/\overline{m}_d)^{1/2}$. The effective dust cyclo-

*Email address: duanws@nwnu.edu.cn

tron frequency is defined by $\omega_{cd}=Be\overline{Z}_{d0}/\omega_{pd}\overline{m}_d$.

The dimensionless variables that we will use are defined below. The ion and electron densities are normalized by $N_{tot}\overline{Z}_{d0}$. The dust density is normalized by N_{tot}. Z_{dj} is normalized by $\overline{Z}_{d0}$. m_{dj} is normalized by $\overline{m}_d$. The space coordinates x,y,z are normalized by λ_{Dd}. The dust velocity $\vec{u}$, time t, and the electrostatic potential ϕ are normalized by C_d, ω_{pd}^{-1}, and T_{eff}/e, respectively. The dust grains are assumed monodispersive, and they emit no electrons. The latter assumption is appropriate for laboratory experiments, but not for astrophysical conditions where photoemission and secondary emission often result in a positive dust charge. By using the normalized variables, we obtain the following equations:

$$\frac{\partial n_{dj}}{\partial t}+\frac{\partial}{\partial x}(n_{dj}u_{dj})+\frac{\partial}{\partial y}(n_{dj}v_{dj})+\frac{\partial}{\partial z}(n_{dj}w_{dj})=0, \quad (1)$$

$$\frac{\partial u_{dj}}{\partial t}+u_{dj}\frac{\partial u_{dj}}{\partial x}+v_{dj}\frac{\partial u_{dj}}{\partial y}+w_{dj}\frac{\partial u_{dj}}{\partial z}=\frac{Z_{dj}}{m_{dj}}\frac{\partial \phi}{\partial x}+\omega_{cd}\frac{Z_{dj}}{m_{dj}}v_{dj}, \quad (2)$$

$$\frac{\partial v_{dj}}{\partial t}+u_{dj}\frac{\partial v_{dj}}{\partial x}+v_{dj}\frac{\partial v_{dj}}{\partial y}+w_{dj}\frac{\partial v_{dj}}{\partial z}=\frac{Z_{dj}}{m_{dj}}\frac{\partial \phi}{\partial y}-\omega_{cd}\frac{Z_{dj}}{m_{dj}}u_{dj}, \quad (3)$$

$$\frac{\partial w_{dj}}{\partial t}+u_{dj}\frac{\partial w_{dj}}{\partial x}+v_{dj}\frac{\partial w_{dj}}{\partial y}+w_{dj}\frac{\partial w_{dj}}{\partial z}=\frac{Z_{dj}}{m_{dj}}\frac{\partial \phi}{\partial z}, \quad (4)$$

where $j=1,2,\ldots,N$, and also

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}=\sum_{j=1}^{N} n_{dj}Z_{dj}+n_e-n_i \quad (5)$$

with $n_i=\mu e^{-s\phi}$ and $n_e=\nu e^{s\beta'\phi}$, where $\beta'=T_i/T_e$, $s=1/(\mu+\nu\beta')$. The Boltzmann distributed ions and electrons are valid for $\omega\ll k_zV_{Te},\omega_{ce}k_z/k_\perp,\omega_{ce}$, and short wavelength compared with the ion gyroradius. The linearization of Eqs. (1)–(5) yields $1+1/(k^2\lambda_D^2)-[\omega_{pd}^2/(\omega^2-\omega_{cd}^2)](k_\perp^2/k^2)-(\omega_{pd}^2/\omega^2)(k_z^2/k^2)=0$, where $\lambda_D=\lambda_{De}\lambda_{Di}/(\lambda_{De}^2+\lambda_{Di}^2)^{1/2}$, and λ_{De} (λ_{Di}) is the electron (ion) Debye radius. In the limit of $\omega\ll\omega_{cd}$, it is found that $\omega=k_zC_d/[1+k_\perp^2(\rho_d^2+\lambda_D^2)+k_z^2\lambda_D^2]^{1/2}$ if $k_\perp^2(\rho_d^2+\lambda_D^2)=k_\perp^2\rho_{dD}^2\ll 1$, and $k_z^2\lambda_D^2\ll 1$, and we find that $\omega\approx k_zC_d(1-\frac{1}{2}k_\perp^2\rho_{dD}^2-\frac{1}{2}k_z^2\lambda_D^2)$. It is clear that dust size distribution modifies $C_d=\omega_{pd}\lambda_d$ (dust acoustic wave velocity) and $\rho_d=\lambda_D\omega pd/\omega_{cd}$ through the modification of ω_{pd} and ω_{cd}, which is different from that reported by Rao *et al.* [5] where they only studied the dusty plasmas composed by monosized dust grains. However they are same for dusty plasmas of monosized dust grains.

III. MATHEMATICAL FORMALISM

In order to study dust acoustic waves, we use a traditional perturbation method. For long wavelength limit $\sqrt{k_\perp^2+k_z^2}\ll 1$, we let ϵ to be about the order of $\sqrt{k_\perp^2+k_z^2}$. We study the small amplitude nonlinear waves which mainly propagate in the z direction. So we only consider the waves whose amplitude of dust particle density variations, electrostatic potential variations, and the dust velocity component in the z direction is about the order of $k_\perp^2+k_z^2$ or about the order of ϵ^2, the dust velocity components in both x and y directions are about the order of $(k_\perp^2+k_z^2)^{3/2}$ or about the order of ϵ^3. Therefore the variables are stretched as follows:

$$\begin{pmatrix}\xi\\ \eta\\ \zeta\\ \tau\end{pmatrix}=\epsilon\begin{pmatrix}x\\ y\\ z-v_0t\\ \epsilon^2 t\end{pmatrix},$$

and

$$\begin{pmatrix}n_{dj}\\ u_{dj}\\ v_{dj}\\ w_{dj}\\ \phi\end{pmatrix}=\begin{pmatrix}n_{dj0}\\ 0\\ 0\\ 0\\ 0\end{pmatrix}+\epsilon^2\begin{pmatrix}n_{dj1}\\ \epsilon u_{dj1}\\ \epsilon v_{dj1}\\ w_{dj1}\\ \phi_1\end{pmatrix}+\epsilon^4\begin{pmatrix}n_{dj2}\\ u_{dj2}\\ v_{dj2}\\ w_{dj2}\\ \phi_{dj2}\end{pmatrix}.$$

Substituting above expansions into Eqs. (1)–(5), we obtain the following results: $w_{dj1}=-(Z_{dj}/v_0m_{dj})\phi_1$, $n_{dj1}=-(n_{dj0}Z_{dj}/v_0^2m_{dj})\phi_1$, $v_0^2=\Sigma_{j=1}^N n_{dj0}Z_{dj}^2/m_{dj}$, $v_{dj1}=-(1/\omega_{cd})(\partial\phi_1/\partial\xi)$, $u_{dj1}=(1/\omega_{cd})(\partial\phi_1/\partial\eta)$, $v_{dj2}=-(v_0/\omega_{cd}^2)(m_{dj}/Z_{dj})(\partial^2\phi_1/\partial\eta\partial\zeta)$, $u_{dj2}=-(v_0/\omega_{cd}^2)(m_{dj}/Z_{dj})(\partial^2\phi_1/\partial\xi\partial\zeta)$, where ϕ_1 satisfies the ZK equation

$$\frac{\partial \phi_1}{\partial \tau}+A\phi_1\frac{\partial \phi_1}{\partial \zeta}+B\frac{\partial^3 \phi_1}{\partial \zeta^3}+C\frac{\partial}{\partial \zeta}\left(\frac{\partial^2 \phi_1}{\partial \xi^2}+\frac{\partial^2 \phi_1}{\partial \eta^2}\right)=0 \quad (6)$$

with $A=-(3/2v_0^3)\Sigma_{j=1}^N(n_{dj0}Z_{dj}^3/m_{dj}^2)-(v_0/2)s^2(\nu\beta'^2-\mu)$, $B=v_0/2$, and $C=(v_0/2)[1+(1/\omega_{cd}^2)\Sigma_{j=1}^N n_{dj0}m_{dj}]$.

It is clear that one-dimensional limit follows when $k_\perp^2\rho_{dD}^2\ll 1$, in which case the effects of dust polarization drift disappear. It is found that both the nonlinearity coefficient A and the dispersive coefficient B are not affected by external magnetic field. It is suggested that for one-dimensional waves propagating in the z direction the magnetic field does not affect the nonlinear waves.

IV. CONTRIBUTION OF DUST SIZE DISTRIBUTION

Now we describe the different dust species in a discontinuous model. We start with two different dust species, with respective grain sizes a_1 and a_2, and densities N_{d1} and N_{d2}.

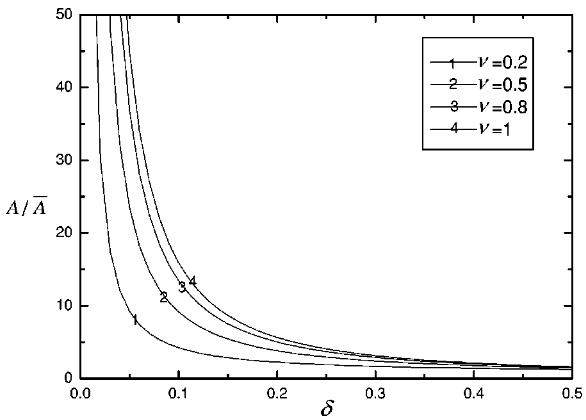

FIG. 1. The variations of $A/\underline{A}$ with respect to δ for different $\nu=0.2,0.5,0.8,1$.

The average size is logically given by $\overline{a}=(N_{d1}a_1+N_{d2}a_2)/(N_{d1}+N_{d2})$. We then define the following quantities: $m_{d1}=m(a_1)\sim a_1^3$, $Z_{d1}=Z_{d1}(a_1)\sim a_1$, $m_{d2}=m(a_2)\sim a_2^3$, $Z_{d2}=Z_{d2}(a_2)\sim a_2$, $\overline{m}_d=m(\overline{a})\sim\overline{a}^3$, and $\overline{Z}_d=Z_d(\overline{a})\sim\overline{a}$. We also define the following velocities: $v_{01}^2=Z_{d1}^2N_{d1}/m_{d1}$, $v_{02}^2=Z_{d2}^2N_{d2}/m_{d2}$, $\overline{v}_0^2=\overline{Z}_d^2(N_{d1}+N_{d2})/\overline{m}_d$. It is noted that the total charge that resides on the dust grains is the same as the case where all dust grains have the same mean radius. We introduce the quantities $\delta=a_1/a_2$, $\nu=N_{d2}/N_{d1}$. To model a realistic power law, we assume that $\delta<1$, $\nu<1$. The following ratio is investigated: $(v_{01}^2+v_{02}^2)/\overline{v}_0^2=1+\nu(1-\delta)^2/\delta(1+\nu)^2>1$. It indicates that two dust species increase the propagation velocity of solitary waves. Of course the same reasoning holds for three or more dust species.

The ZK equation (6) also has a one-dimensional stationary solitary wave solution as follows: $\phi_{10}=\phi_m\mathrm{sech}^2[\zeta-u_0\tau/w]$, where $\phi_m=3u_0/A$ and $w=2\sqrt{B/u_0}$ are the amplitude and width of the soliton, respectively. For a typical dusty plasma [22], we estimate that the second term of A can be neglected compared with first term. Therefore, we obtain

$$A\approx-\frac{3}{2v_0^3}\sum_{j=1}^{N}\frac{n_{dj0}Z_{dj}^3}{m_{dj}^2}. \tag{7}$$

It is clear that $A<0$, $B>0$, and $C>0$, therefore the soliton is rarefactive. We now study a dusty plasma with two different dust species. Then the ratio of $A/\underline{A}$ is $A/\underline{A}=(1+\nu\delta^3/1+\nu)[(\nu+\delta)/(\delta+\nu\delta^2)]^{3/2}$, where A and $\underline{A}$ are the coefficients of the nonlinear term in ZK equation (6) for different dust grains and average dust grains, respectively. Figure 1 shows the variations of $A/\underline{A}$ with respect to ν and δ. It seems that $A/\underline{A}\geqslant 1$. We also find that $A/\underline{A}$ increases as ν increases and decreases as δ increases.

For monosized dusty plasma, the change of polarity is well known in the literature [23,24]. It has been observed by the dust detector on the Ulysses spacecraft during its Jupiter encounter that for very small grains in the size range $0.03\ \mu\mathrm{m}<a<0.14\ \mu\mathrm{m}$, the grains are all positively charged due to secondary electron emission [13]. For this case it is easy to note that the solitary waves are compressive since $A>0$ which is an interesting result. It is also known that the secondary electron emission from spherical grains can lead to oppositely charged dust grains either due to different grains following different charging histories or due to different dust size [13,14]. We now assume that there are two kinds of dust grains, where $a_1<a_2$. If they are all positively charged, which is not a usual case, then there are compressive solitary waves. For another case based on the experimental results [13,14], we assume that there are two different dust grains which are oppositely charged. For smaller dust grains $Z_{d1}<0$ which are positively charged, and for larger ones $Z_{d2}>0$ which are negatively charged. Then we have the following equations:

$$\breve{v}_0^2=\frac{N_{d1}|Z_{d1}|^2}{k_m a_1^3}+\frac{N_{d2}|Z_{d2}|^2}{k_m a_2^3}, \tag{8}$$

$$\breve{A}=\frac{3}{2\breve{v}_0^3}\left(\frac{N_{d1}|Z_{d1}|^3}{(k_m a_1^3)^2}\right)\left[1-\left(\frac{a_1}{a_2}\right)^6\frac{N_{d2}}{N_{d1}}\left|\frac{Z_{d2}}{Z_{d1}}\right|^3\right]. \tag{9}$$

It is easy to note that $\breve{v}_0^2$ is always positive. But the sign of $\breve{A}$ is determined by the sign of $A_s=1-(a_1/a_2)^6(N_{d2}/N_{d1})|Z_{d2}/Z_{d1}|^3$. $A_s>0$ means that $(N_{d2}/N_{d1})|Z_{d2}/Z_{d1}|^3<(a_2/a_1)^6$. For typical dust grains reported in Ref. [13], $a_1=0.01\ \mu\mathrm{m}$ and $a_2=0.1\ \mu\mathrm{m}$, so there are compressive solitary waves when the inequality $(N_{d2}/N_{d1})|Z_{d2}/Z_{d1}|^3<10^6$ is satisfied.

V. MULTIDIMENSIONAL SOLITARY WAVE SOLUTION

In this section we study a multidimensional solitary wave solution of Eq. (6). In order to do this, we assume that there is a solution of $\phi_1=\phi_1(X)$, where $X=\zeta+\lambda_1\xi+\lambda_2\eta-\omega'\tau$. Then we obtain that there is one soliton solution as follows [25]:

$$\phi_1=(3\omega'/A)\mathrm{sech}^2\left(\tfrac{1}{2}\sqrt{\omega'/[B+C(\lambda_1^2+\lambda_2^2)]}(X-X_0)\right),$$

where we have assumed that ϕ_1 and all its derivatives tend to zero when $X\to\infty$, and X_0 is a constant. The soliton amplitude and width are $3\omega'/A$ and $2\sqrt{[B+C(\lambda_1^2+\lambda_2^2)]/\omega'}$, respectively. For continuous power law distribution [11,12], we find that $v_0^2/\overline{v}_0^2=[(\beta-1)^2(c^{-\beta}-1)(c^{2-\beta}-1)]/[\beta(\beta-2)(c^{1-\beta}-1)^2]$ and $A/\overline{A}=[\overline{v}_0^3(\beta-1)^4(c^{-\beta-2}-1)(c^{2-\beta}-1)^3]/v_0^3(\beta+2)(\beta-2)^3(c^{1-\beta}-1)^4$, where $\overline{v}_0$ and $\overline{A}$ are those with the average dust size of the power law distribution, respectively. $c=a_{max}/a_{min}$, where a_{max} and a_{min} are the maximum and the minimum dust size for the power law distribution, respectively. C can be rewritten as $C=v_0/2+(v_0/2\omega_{pd}^2)c'$, where $c'=\Sigma_{j=1}^{N}n_{dj0}m_{dj}$. Then we obtain

$$\frac{c'}{\underline{c'}}=\frac{(\nu+\delta^3)(1+\nu)^2}{(\nu+\delta)^3} \tag{10}$$

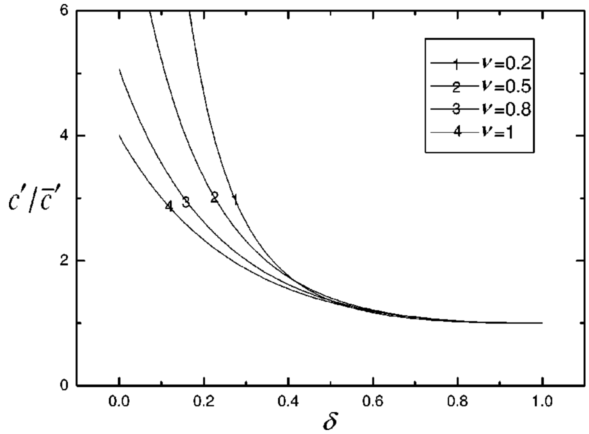

FIG. 2. The variations of $c'/\underline{c'}$ with respect to δ for different $\nu = 0.2, 0.5, 0.8, 1.0$.

for two different dust size distributions, where $\underline{c'}$ is that for monosized dust grains. Figure 2 shows the variations of $c'/\underline{c'}$ with respect to ν and δ. It seems that $c' > \underline{c'}$. For the case of a continuous power law distribution, we find that

$$\frac{c'}{\underline{c'}} = \frac{2-\beta}{4-\beta} \frac{(c^{4-\beta}-1)(c^{1-\beta}-1)^2}{(c^{2-\beta}-1)^3}. \tag{11}$$

Figure 3 shows the variations of $c'/\underline{c'}$ with respect to c for different β. It is noted then that different dust size distribution can let the coefficient C of ZK equation (6) become large. For saturn F ring the grain radii ranged from 0.5 μm to 10.0 μm and followed a power law distribution with $\beta = 3.5$ [26]. It is noted that v_0^2 is ≈ 1.2 times that for averaged monosized dusty plasma. Moreover A and B are about 1.5 and 1.1 times that for monosized dusty plasmas with average dust grains.

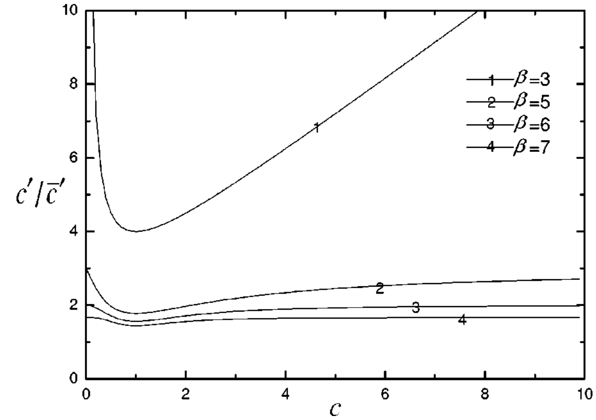

FIG. 3. The variations of $c'/\underline{c'}$ with respect to c for different $\beta = 3,5,6,7$.

We now conclude that, for a dusty plasma composed of different dust grains, different dust size distributions can let both the soliton velocity and the width become larger, while the soliton amplitude becomes smaller.

We conclude that if there are positively charged dust grains which are usually smaller ones, the values of the soliton velocity as well as the soliton width may be very large, and the soliton amplitude may be very small. The possibility of collisions among the dust grains will be much increased. It will increase the possibility of the accretion of the dust grains due to collisions and Coulomb attractions. This is an interesting result for both laboratory and space dusty plasmas.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 10247008) and partially supported by the Foundation of The Royal Society of the U.K.

[1] C.K. Goertz, Rev. Geophys. **27**, 271 (1989).
[2] O. Havnes *et al.*, Phys. Scr. **45**, 433 (1992).
[3] S. Benkadda, V.N. Tsytovich, and A. Verga, Comments Plasma Phys. Controlled Fusion **16**, 321 (1995).
[4] F. Verheest, Space Sci. Rev. **77**, 267 (1996).
[5] N.N. Rao *et al.*, Planet. Space Sci. **38**, 543 (1990).
[6] P.K. Shukla and V.P. Silin, Phys. Scr. **45**, 508 (1992).
[7] A. Barkan *et al.*, Phys. Plasmas **2**, 3563 (1995).
[8] G.E. Morfill *et al.*, J. Vac. Sci. Technol. A **14**, 490 (1996).
[9] R.L. Merlino *et al.*, Planet. Space Sci. **38**, 1143 (1990).
[10] N. D'Angelo, Planet. Space Sci. **38**, 1143 (1990).
[11] P. Meuris, Planet. Space Sci. **45**, 449 (1997).
[12] A. Brattli *et al.*, J. Plasma Phys. **58**, 691 (1997).
[13] V.W. Chow, D.A. Mendis, and M. Rosenberg, J. Geophys. Res., [Planets] **98**, 19065 (1993).
[14] M. Horanyi and C.K. Goertz, Astrophys. J. **361**, 155 (1990).
[15] P. Meuris, Planet. Space Sci. **45**, 1171 (1997).
[16] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, 2000).
[17] P. Bliokh *et al.*, Plasma Phys. Rep. **22**, 411 (1996).
[18] F. Verheest *et al.*, Phys. Plasmas **9**, 2479 (2002).
[19] F. Verheest *et al.*, Phys. Scr. **64**, 494 (2001).
[20] V.V. Yaroshenko *et al.*, Phys. Rev. E **64**, 036401 (2001).
[21] A.A. Mamun, Phys. Scr. **58**, 505 (1998).
[22] A. Barkan *et al.*, Phys. Rev. Lett. **73**, 3093 (1994).
[23] P.H. Sakanaka and P.K. Shukla, Phys. Scr., T **T84**, 181 (2000).
[24] P.H. Sakanaka *et al.*, Br. J. Pharmacol. **33**, 158 (2003).
[25] M. Wadati, J. Phys. Soc. Jpn. **38**, 673 (1975).
[26] D.A. Mendis, and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1996).